Phil 2310
Fall 2010

Assignment 6: This homework is due by the beginning of class on Fri, Oct 15th.

Part I: Practicing Taut Con

Show that each of the following arguments is valid by constructing a proof in $\mathcal{F}$. You should write out each proof on a piece of paper and hand it in to me in class. You could also write your proof in Fitch and then simply print it out or email them to me. If you do that, you must click 'show step numbers' and also 'verify proof' before you print it.

You may use any rules of $\mathcal{F}_T$ plus you can use Taut Con for any step that I consider to be sufficiently obvious. This will be a judgment call so err on the side of caution (and use other rules). If it is something we explicitly mentioned in class, that is okay. Below are other steps that are okay uses of Taut Con. Some of these will be helpful for the problems and the problems are written to get you to use some of these.

Modus Tollens
$P \to Q, \neg Q \vdash \neg P$

Conditionals
$\neg P \vee Q \Leftrightarrow P \to Q$
$P \wedge \neg Q \Leftrightarrow \neg(P \to Q)$

Disjunctive Syllogism
$P \vee Q, \neg P \vdash Q$

DeMorgan's Laws
$\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$
$\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$

Biconditionals
$P \leftrightarrow Q, \neg P \vdash \neg Q$
$P \leftrightarrow Q \Leftrightarrow \neg P \leftrightarrow \neg Q$
$\neg(P \leftrightarrow Q) \Leftrightarrow \neg P \leftrightarrow Q$

1. $P \vee Q \vdash (\neg Q \to \neg P) \to Q$
2. $P \to Q, \neg P \to R \vdash Q \vee R$
3. $\neg(P \wedge Q), \neg(\neg P \wedge Q) \vdash \neg Q$
4. $(P \to Q) \to P, P \to R \vdash R$
5. $(P \to Q) \to Q \vdash (Q \to P) \to P$
6. $(P \to Q) \vee (R \to S) \vdash (P \to S) \vee (R \to Q)$
7. $P \leftrightarrow (Q \leftrightarrow R) \vdash (\neg Q \wedge \neg P) \to R$

Part II. Write a sentence using only $\neg$ and $\vee$ as connectives that is equivalent to the following sentence: $(P \to Q) \leftrightarrow (\neg S \wedge R)$

[Hint – think about some of the above equivalences]

Part III: Propositional Logic Metatheory

1. Consider the made-up rule ∨IA:
   (for ∨Intro in the antecedent)

$$\begin{array}{|ll} \text{k. } X\to Z & \\ \hline \text{l. } (X\lor Y)\to Z & \lor\text{IA k.} \end{array}$$

Would this be an acceptable short-cut rule to add to our proof system? Why or why not? Would the Soundness and Completeness Theorems still be true of this new system?

2. Consider the made-up rule ∨CA:
   (for ∨ chain argument)

$$\begin{array}{|ll} \text{k. } X\lor Y & \\ \text{m. } Z\lor\neg Y & \\ \hline \text{n. } X\lor Z & \lor\text{CA k,m} \end{array}$$

Would this be an acceptable short-cut rule to add to our proof system? Why or why not? Would the Soundness and Completeness Theorems still be true of this new system?

Part IV.
Read Chapter 9 in our book.
Do problems 9.16 and 9.17 (lots of translations)

ANSWERS

For part I, any 'reasonable' use of Taut Con is allowed. Obviously, the line they get actually has to be a consequence of the lines they cite. Don't take points off unless it really looks like they were just using guess and check. For example, if they just do a 1 line proof going from the premises to the conclusion by Taut Con.

For part II, first, note that I accidentally had '~' instead of '¬' in the question initially, so don't be surprised to see it (I wonder who will use '~' because they think that was intended?) Anything equivalent to the original sentence is acceptable. Here are some steps to get to possible answers:

$(P\to Q) \leftrightarrow (\neg S \land R)$ [[first remove biconditional]] =
$[(P\to Q) \to (\neg S \land R)] \land [(\neg S \land R) \to (P\to Q)]$ [[then DeMorgan's] =
$\neg(\neg[(P\to Q) \to (\neg S \land R)] \lor \neg[(\neg S \land R) \to (P\to Q)])$ [[then remove arrows]] =

¬(¬[¬(¬P∨Q) ∨ (¬S ∧ R)] ∨ ¬[¬(¬S ∧ R) ∨ (¬P∨Q)]) [[then remove conjunctions]] =
¬(¬[¬(¬P∨Q) ∨ ¬(S ∨ ¬R)] ∨ ¬[(S ∨ ¬R) ∨ (¬P∨Q)])

If they do biconditional differently:
(P→Q) ↔ (¬S ∧ R) [[first remove biconditional]] =
[(P→Q) ∧ (¬S ∧ R)] ∨ [¬(P→Q) ∧ ¬(¬S ∧ R)] [[then arrows]] =
[(¬P∨Q) ∧ (¬S ∧ R)] ∨ [¬(¬P∨Q) ∧ ¬(¬S ∧ R)] [[then internal conjunctions]] =
[(¬P∨Q) ∧ ¬(S ∨ ¬R)] ∨ [¬(¬P∨Q) ∧ (S ∨ ¬R)] [[then big conjunctions]] =
¬[¬(¬P∨Q) ∨ (S ∨ ¬R)] ∨ ¬[(¬P∨Q) ∨ ¬(S ∨ ¬R)]

If they do biconditional last:
(P→Q) ↔ (¬S ∧ R) [[remove internal parts]] =
(¬P∨Q) ↔ ¬(S ∨ ¬R) [[turn biconditional to arrows]] =
[(¬P∨Q) → ¬(S ∨ ¬R)] ∧ [¬(S ∨ ¬R) → (¬P∨Q)] [[then arrows]] =
[¬(¬P∨Q) ∨ ¬(S ∨ ¬R)] ∧ [(S ∨ ¬R) ∨ (¬P∨Q)] [[then conjunction]] =
¬(¬[¬(¬P∨Q) ∨ ¬(S ∨ ¬R)] ∨ ¬[(S ∨ ¬R) ∨ (¬P∨Q)])

Actually, by glancing at student homeworks, it now occurs to me that there is an obvious way to check their answer if you aren't sure. Open fitch, enter my sentence on line 1 and their sentence on line 2 justified by taut con 1. Fitch will tell you if this is okay. To make sure it is equivalent and not just a consequence, make sure the original sentence follows from what they have as well.

For part III the first one Soundness would fail, the second one Soundness would be fine. For number 1, they should say enough so that it is clear that they recognize that the argument is invalid and they should say enough so that it is clear that they know why that matters (this new proof system could prove invalid arguments, but soundness entails that you can prove only valid arguments).

An awesome answer for 1) This rule would not be an acceptable short cut rule to add to our proof system. The new system would still be complete since adding rules means that you can still prove all of the valid arguments you could before (just now you might be able to prove more). But the new system would not be sound. The soundness theorem would fail since ∨IA can introduce the first invalid step in a proof. An example of this is a two line proof were line 1 is X→Z and line 2 is (X∨Y)→Z. Since line 1 depends only on itself, it is a valid step. Line 2 depends on line 1 which is still an active premise. But the sentence on the line is not tf-entailed by line 1. This can be seen by giving an invalidating assignment: X:F Y:T Z:F. Thus it is possible for our new system to prove arguments that are not really valid hence the Soundness Theorem would be false of this system.

Note that they definitely don't need to say this much

Awesome answer for 2) This rule would be an acceptable rule to add to our proof system. The new system would still be complete since adding rules means that you can still prove all of the valid arguments you could before (just now you might be able to prove more). The system would still be sound as well. Since we know that our old system is sound, the new system is sound if and only if the new rule ∨CA is not capable of introducing the first invalid step in a proof. And it can't do this. For Reductio purposes, assume that it could introduce the first invalid step. Then in some proof, there would be a line of the form $X \vee Z$ which is an invalid step which means that it is not tf-entailed by the undischarged assumptions which it depends on. Since any example of ∨CA means that you have sentences of the form $X \vee Y$ and $X \vee \neg Y$ on earlier lines, and since we are assuming line n. is the first invalid step, we know that our earlier lines k. and m. are valid steps. Line n. depends on all of the assumptions (A1, A2, A3…) that either of the earlier lines depend on (and possibly more). Since it is an invalid step, we know that there is a truth value assignment making all of (A1, A2, A3…. and possibly more) true and making $X \vee Z$ false. Since this assignment makes $X \vee Z$ false, it must make X false and Z false. Since any assignment which makes (A1, A2, A3…. and possibly more) true also makes all of the assumptions that line k. depends on true (since this is a smaller set included in the larger one) this assignment must make $X \vee Y$ true. Since it makes X false, it makes Y true. But any assignment which makes (A1, A2, A3…. and possibly more) true also makes all of the assumptions that line m. depends on true (since this is also a smaller set included in the larger one) this assignment must make $Z \vee \neg Y$ true. Since it makes Z false, it makes $\neg Y$ true and hence Y false. So any such invalidating assignment makes Y true and Y false. But this is a contradiction hence there is no such assignment hence $X \vee Z$ really does follow from the assumptions that line k. depends on hence the rule ∨CA cannot introduce the first invalid step in a proof hence the new proof system would still be sound.

Note that they definitely don't need to say this much. I was not explicit when I told them how to answer these questions so if they answer in a way that doesn't even mention anything about the assumptions that the line depends on, that is fine. This would be okay if none of the rules messed with subproofs. But I won't ask them about any questions which mess with scope lines, so they can ignore it.

For overall grading, the translations in part IV should be worth a total of 30 points. 1 point each. Sign on to the grade grinder and look if people submitted and if they did and they are 'green' they got them all correct. If red, just look to see how many they got wrong.

Lets make the first part 7 pts each (so out of 49) and then 7 pts each for part II and each question on part III.